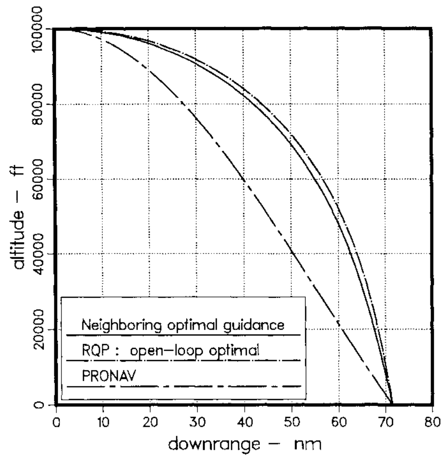

Fig. 2 Altitude downrange comparisons.

At each sample point, the value of $\bar{M}$ is updated from inertial measurements.

To demonstrate the effectiveness of this guidance law, it has been flown in a three-degree-of-freedom simulation containing a true time-based physical flight model, a standard atmosphere, and wind-tunnel based aerodynamics.[5] Results are shown in Fig. 2 for a sample time of $\Delta t = 0.1$ s.

Also shown in these figures are the open-loop optimal trajectory for the simulation model and the trajectory obtained from proportional navigation (PRONAV). The open-loop optimal trajectory has been obtained for a piecewise-linear control (suboptimal control) using recursive quadratic programming (RQP).[6] The PRONAV guidance law is that of the linear-quadratic control law of Ref. 7. Weighting is for miss distance only, leading to the familiar control gain, $K = 3/t_{go}^2$, where t_{go} is calculated at each sample point as (range/range rate).

A fourth-order, fixed step, Runge-Kutta integrator is used in all simulations. The nominal trajectory boundary conditions (initial-final) for the example problem are: $\gamma = 0$ deg-free, $x = 0$ nm-71.51 (the final downrange is that predicted by the proposed guidance scheme at the initial value of $\bar{M}$), h = 100,000 ft-0, V = 11,000 ft/s-max.

Figure 2 displays the simulation flight profiles. The neighboring optimal guidance and RQP have the same general glide-and-dive contour, vs PRONAV, which turns quickly to line up with the target. These first two simulations shift the majority of flight time to the higher altitudes, where drag is low.

The true optimal velocity produced by the RQP scheme is 7090 ft/s for a flight time of 45 s. The terminal velocity generated by the proposed method differs from it by less than 1%, whereas the PRONAV solution is about 40% less, with a 10% increase in flight time. Because they take advantage of the atmospheric density variation during descent, lift coefficient variation is moderate for the optimal scheme, vs PRONAV, which applies maximum turning from the start to line up with the target.

The issue of robustness was addressed by examining the ability to hit the nominal target (i.e., 71.51 nm downrange) given changes in initial velocity and downrange. Velocity perturbations of ±2000 ft/s and downrange perturbations of ±10 nm were used. In each case, the vehicle was able to hit the target. For a given initial velocity, terminal velocity results appear relatively insensitive to initial downrange perturbations.

Conclusions

A sampled-data feedback control method has been devised to obtain approximate, maximum-terminal-velocity descent trajectories in a vertical plane at a designated target. These trajectories are characterized by glide-and-dive flight to the target to minimize the time spent in the denser parts of the atmosphere. The proposed on-line scheme uses neighboring optimal theory to successfully bring final altitude and range constraints together, as well as compensate for differences in flight model, atmosphere, and aerodynamics. Comparison with the open-loop optimal trajectory for the terminal velocity is excellent and far exceeds the proportional navigation solution. The scheme also demonstrates robustness to significant perturbations in initial velocity and downrange.

Acknowledgment

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Factorization Approach to Control System Synthesis

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Nomenclature

$a(s)$ = $a(s) \in \Psi$
$b(s)$ = $b(s) \in \Psi$
$D(s)$ = $D(s) \in H$; the pair $[N(s), D(s)] \in H$ are coprime factors of $h_{y,u}^p$
H = commutative algebraic ring; $H \subset R(s)$ with Hurwitz denominator polynomials
h = altitude; 30,000 ft
$h_{y,u}$ = closed-loop input-output map
$h_{y,u}^D$ = desired closed-loop input-output map
$h_{y,u}^p$ = plant input-output map
M = Mach number; 0.84
$m(s)$ = $m(s) \in \Psi'$
$N(s)$ = $N(s) \in H$; see $D(s)$
n_r = number of unknown coefficients on the right-hand side of the Bezout identity
$P(s)$ = $P(s) \in H$; the pair $[P(s), Q(s)] \in H$ is coprime and satisfies the Bezout identity

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$p_d(s) = p_d(s) \in \Psi'$
$p_n(s) = p_n(s) \in \Psi$
$Q(s)$ = $Q(s) \in H$; see $P(s)$
$q_d(s) = q_d(s) \in \Psi'$
$q_n(s) = q_n(s) \in \Psi$
$R(s)$ = set of all rational functions formed by the ratio of two polynomials in $s = j\omega$ with real coefficients
$r(s)$ = function parameter; $\in H$
s = Laplace variable
u = plant excitation
x_i = coefficients of the unknown polynomial $q_n(s)$; subscripted for enumeration
y = output; aircraft pitch rate
y_j = coefficients of the unknown polynomial $p_n(s)$; subscripted for enumeration
δ = degree of a polynomial; may be subscripted for clarity
δ_1 = sum of δ_m and δ_d
Ψ = set of all polynomials with real coefficients
Ψ' = subset of Ψ with Hurwitz elements
Ξ = set of all stabilizing compensators
ω = frequency

Introduction

IN the following discourse, a new technique for design synthesis of automatic control systems with a factorization infrastructure is presented. The design technique is a solution to the closed-loop model matching problem, and at present it is applicable to linear, univariate, deterministic, time-invariant, minimum phase-nonminimum phase, proper-improper, and stable as well as unstable systems. When applying the stable factorization techniques, it is not clear how one should search the solution space of the *function-parameter* $r(s)$ not only to assure stability but also to achieve certain performance measures. The main contribution of the work presented herein is the development of a possible search technique, and this contribution is made in the context of the factorization theory. This problem in its original setting has not been solved as we have directly done so here. More importantly, our solution space is described in terms of a set of linear simultaneous algebraic equations. The procedure uses a slightly modified version of a recently developed algorithm for the solution the Bezout identity[1] to render the usual tendency for design of high-order compensators, which is common with various algebraic design techniques. The technique is applied to an aircraft control problem studied earlier[2]; application of the technique to other paradigms[3,4] revealed similar characteristics.

In the ring of polynomials, a factorization approach for compensator design purposes was developed in Ref. 4, and the method was demonstrated by designing a compensator for a popular and difficult to control plant that was also later studied by other researchers (e.g., Ref. 5). The coefficients of a cascade compensator by minimizing a weighted mean-square error function was developed in Ref. 2. However, unlike our formulation (which is based on the Youla parameterization[6]), that technique is not applicable to unstable systems. There has also been recent developments in compensator design using H_2/H_∞ (Ref. 7) and other related techniques, such as the ad hoc μ synthesis technique[8]; however, the framework of the pertaining body of that literature is intended to solve a similar but different problem.

Formulation

Parameterization of the class of all compensators for plant $h^p_{y,u}$ that place system closed-loop poles in a specified region of the s plane for any $r(s) \in H$ is given by the following expression:

$$\Xi := \left\{ \frac{P(s) - r(s)D(s)}{Q(s) + r(s)N(s)},\ r(s) \in H \right\} \tag{1}$$

The as yet unknown function parameter $r(s)$ may be selected to achieve certain performance criteria. The following algorithm is used to solve for the coprime pair $[P(s), Q(s)] \in H$.

Step 1: Form $m(s) \in H$; the least allowable degree of $m(s)$, δ_m, is equal to the max $\{\delta_a, \delta_b\}$ to assure the properness of both $N(s)$ and $D(s)$.

Step 2: Determine the degree of $p_d(s)$, δ_{p_d}, and the degree of $q_d(s)$, δ_{q_d}, by letting $\delta_{p_d} = \delta_{q_d} = \delta_d$ where $\delta_d = \delta_m - 1$.

Step 3: Select the poles of $[P(s), Q(s)]$ within the algebraic ring H and form $d(s)$.

Step 4: If $\delta_b > \delta_a$, then let

$$q_n(s) = \sum_{i=0}^{\delta_1 - \delta_b} y_i s^i \tag{2}$$

$$p_n(s) = \sum_{i=0}^{\delta_b - 1} x_i s^i \tag{3}$$

Otherwise, let

$$q_n(s) = \sum_{i=0}^{\delta_a - 1} y_i s^i \tag{4}$$

$$p_n(s) = \sum_{i=0}^{\delta_1 - \delta_a} x_i s^i \tag{5}$$

In this manner, the right-hand side of the result as well as the left-hand side of the Bezout identity results in a polynomial of degree δ_1 with n_r coefficients; the coefficients of the same powers of s are equated, and the unknown coefficients are solved from the resulting simultaneous linear algebraic equations. Lower-order coprime factors may be obtained by selecting $m(s)$ in step 1 to enforce either $N(s)$ or $D(s)$ to have left-half plane pole-zero cancellations.

The following approach is developed to select that $r(s) \in H$ that is used to form a stable closed-loop system having the characteristics of a user-defined closed-loop transfer function.

Compensator Synthesis

In terms of the coprime factors, the input-output map is given by the following equation:

$$h_{y,u} = -N(s)D(s)r(s) + N(s)P(s) = Z_1/Z_2 \tag{6}$$

Then the equation error may be formed and minimized. In Ref. 9 a least-squares error criteria was used; as an alternative approach, a mean-square error function is considered.

$$J = \int_{\omega_1}^{\omega_2} |h_{y,u} - h^D_{y,u}|^2 \,\mathrm{d}\omega \tag{7}$$

Let

$$N(j\omega) = \frac{a_1' + b_1' j}{a_2' + b_2' j} \tag{8}$$

$$D(j\omega) = \frac{c_1 + d_1 j}{c_2 + d_2 j} \tag{9}$$

$$r(j\omega) = \frac{\sum_{i=0}^{i=m} a_i (j\omega)^i}{1 + \sum_{i=1}^{i=n} b_i (j\omega)^i} \tag{10}$$

$$P(j\omega) = \frac{e_1 + f_1 j}{e_2 + f_2 j} \tag{11}$$

$$h^D_{y,u} = \frac{N_1 + N_2 j}{P_1 + P_2 j} = \frac{Y_1}{Y_2} \tag{12}$$

By setting $\nabla J_{a,b} = 0$, a set of nonlinear simultaneous equations will be obtained, and one may choose to apply nonlinear optimization approaches to solve for the unknowns. Here, rather than minimizing Eq. (6), i.e., the quantity $\int_\omega |(Z_1 Y_2 - Z_2 Y_1)/Z_2 Y_2|^2 \, d\omega$, the quantity $\int_\omega |Z_1 Y_2 - Z_2 Y_1|^2 \, d\omega$ is minimized. Note that minimum value of this quantity will always be zero if a solution exists. Therefore, under optimality conditions $Z_1 Y_2 = Z_2 Y_1$, provided that polynomials on both sides of this equation are of the same order. If a particular design calls for a high-order compensator, minimizing $\int_\omega |Z_1 Y_2 - Z_2 Y_1| \, d\omega$ may result in unacceptable dynamics. In such cases, the equivalent desired return ratio should be modified by addition of the necessary high-order dynamic terms. Therefore, upon substitution of Eqs. (6) and (8-12) into Eq. (7) and multiplying the resulting integrand by its common denominator, we obtain the following equation.

$$J' = \int_{\omega_1}^{\omega_2} \left[(-AA_1 + BA_2 + B_1 C - B_2 D)^2 + (-AA_2 - BA_1 + B_1 D + B_2 C)^2\right] d\omega \tag{13}$$

where

$$A_1 = a_0 - a_2\omega^2 + a_4\omega^4 - \cdots \tag{14}$$

$$A_2 = a_1\omega - a_3\omega^3 + a_5\omega^5 - \cdots \tag{15}$$

$$B_1 = 1 - b_2\omega^2 + b_4\omega^4 - \cdots \tag{16}$$

$$B_2 = b_1\omega - b_3\omega^3 + b_5\omega^5 - \cdots \tag{17}$$

$$A = (a_1' - b_1' d_1)(e_2 P_1 - f_2 P_2) - (a_1' d_1 + b_1' c_1)(e_2 P_2 + f_2 P_1) \tag{18}$$

$$C = E - G \tag{19}$$

$$D = F - H \tag{20}$$

$$E = (a_1' - b_1' f_1)(c_2 P_1 - d_2 P_2) - (a_1' f_1 + b_1' e_1)(c_2 P_2 + P_1 d_2) \tag{21}$$

$$F = (a_1' e_1 - b_1' f_1)(c_2 P_2 + P_1 d_2) + (a_1' f_1 + b_1' e_1)(c_2 P_1 - d_2 P_2) \tag{22}$$

$$G = (a_2' c_2 - b_2' d_2)(e_2 N_1 - f_2 N_2) - (a_2' d_2 + b_2' c_2)(e_2 N_2 + f_2 N_1) \tag{23}$$

$$H = (a_2' c_2 - b_2' d_2)(e_2 N_2 + f_2 N_1) - (a_2' d_2 + b_2' c_2)(e_2 N_1 + f_2 N_2) \tag{24}$$

Next, let $\nabla J'_{a,b} = 0$. A set of linear algebraic equations is then obtained. These equations in matrix form are given below.

$$XY = Z \tag{25}$$

where

$$X = \begin{bmatrix} T_0 & 0 & -T_2 & 0 & \cdots & R_1 & S_2 & -R_3 & -S_4 & \cdots \\ 0 & T_2 & 0 & -T_4 & \cdots & -S_2 & R_3 & S_4 & -R_5 & \cdots \\ 0 & -T_4 & 0 & T_6 & \cdots & S_4 & -R_3 & -S_6 & R_7 & \cdots \\ \vdots & & & & \cdots & & & & & \cdots \\ R_1 & -S_2 & -R_3 & S_4 & \cdots & Q_2 & 0 & -Q_4 & 0 & \cdots \\ S_2 & R_3 & -S_4 & -R_5 & \cdots & 0 & Q_4 & 0 & -Q_6 & \cdots \\ -R_3 & S_4 & R_5 & -S_6 & \cdots & -Q_4 & 0 & Q_6 & 0 & \cdots \\ -S_4 & -R_5 & S_6 & R_7 & \cdots & 0 & -Q_6 & 0 & Q_8 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \tag{25a}$$

$$Y = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ \vdots \end{bmatrix}, \quad Z = \begin{bmatrix} S_0 \\ R_1 \\ -S_2 \\ -R_3 \\ \vdots \\ 0 \\ Q_2 \\ 0 \\ Q_4 \\ \vdots \end{bmatrix} \tag{25b}$$

$$T_l = \int_{\omega_1}^{\omega_2} \omega^l (A^2 + B^2) \, d\omega \tag{25c}$$

$$R_l = \int_{\omega_1}^{\omega_2} \omega^l (AD - BC) \, d\omega \tag{25d}$$

$$S_l = \int_{\omega_1}^{\omega_2} \omega^l (AC + BD) \, d\omega \tag{25e}$$

$$Q_l = \int_{\omega_1}^{\omega_2} \omega^l (C^2 + D^2) \, d\omega \tag{25f}$$

The compensator design procedure is summarized in Fig. 1. The procedure is demonstrated by solving an example problem.

Demonstration Example Problem

Consider the pitch rate transfer function of the F-104 aircraft at $M = 0.84$ and $h = 30{,}000$ ft.

$$h^p_{y,u} = \frac{-17.8(s + 0.0144)(s + 0.432)s}{(s^2 + 1.12056s + 12.1104)(s^2 + 0.010404s + 0.002601)} \tag{26}$$

From the flying qualities point of view, the desired closed-loop transfer function follows [Eqs. (26) and (27) are taken from Ref. 2].

$$h^D_{y,u} = \frac{-4.4(s + 0.432)}{s^2 + 5.6s + 16} \tag{27}$$

Since the plant input-output map is stable, we let $N(s) = G(s)$, $D(s) = 1$, $P(s) = 1$, and $Q(s) = 0$. Using Eq. (31) with $\omega_1 = 3.5$ and $\omega_2 = 4.5$, the parameter function $r(s)$ is found to be of the following form.

$$r(s) = \frac{-0.18676 - 0.01729220s - 0.0154213s^2}{1.0 + 0.34931s + 0.0624025s^2} \tag{28}$$

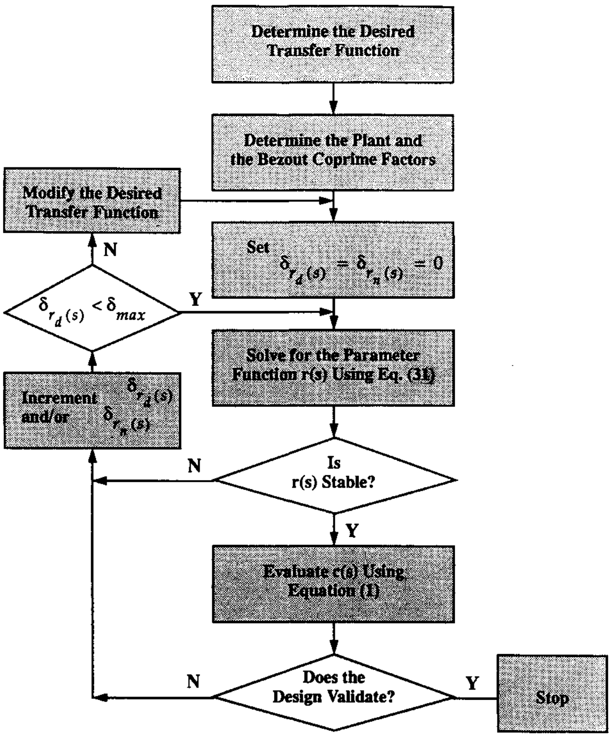

Fig. 1 Design synthesis procedure.

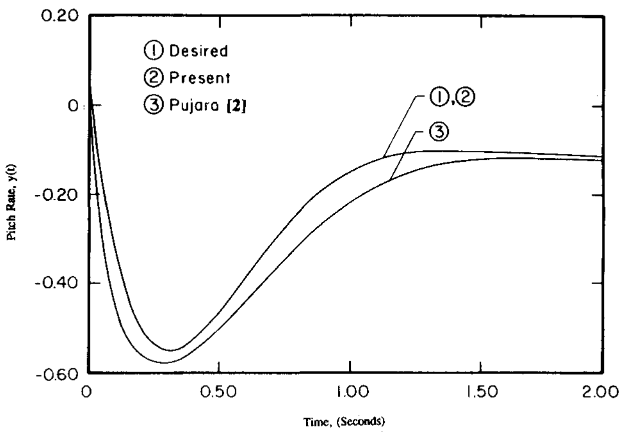

Fig. 2 Unit-step response comparisons for demonstration example problem.

Using Eq. (1), the corresponding compensator will then be given by the following expression.

$$C(s) = \frac{0.18677 + 0.0172919s + 0.0154213s^2}{1.1197 + 0.62381s + 0.06240s^2} \tag{29}$$

The unit step responses are depicted in Fig. 2, and it is apparent that our design meets the objectives more closely than that previously obtained.

Summary and Conclusions

The solution to the closed-loop model-matching problem based on a factorization approach requires one to search the set of all admissible function parameters. A possible search procedure was presented, and the solution space was given in terms of a set of linear algebraic equations, i.e., Eq. (25). The procedure is computer-assisted, and it is based on minimizing a mean-square error criteria.

The procedure may be extended to account for minimizing the sensitivity function as well as to account for integrity conditions. In both cases, the solution space will remain linear.

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Parametric Study of Adaptive Generalized Predictive Controllers

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Introduction

OPTIMAL tracking problems with finite and moving horizons have been studied extensively by Kishi.[1] Brickner and Brogan[2] proposed a controller model that included adaptive model estimation and predictive control to adjust nominal control inputs. The combination of the model identification, predictive controller, and the finite control horizon is essential to what is now called the generalized predictive controller extensively reported by Clarke.[3-9] This Note presents results of a parametric study of generalized predictive control (GPC), based largely on the thesis of Han.[10]

Generalized Predictive Control

The GPC controller minimizes a quadratic cost function over a receding horizon to calculate future control inputs. The system is modeled as

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) + e(t) \tag{1}$$

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